

Review Exam II, MTH 221 , Spring 2011

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QUESTION 1. Let $F = span\{1 - x + x^2, -1 + x^2, 2x\}$

(i) Find a basis for F.

(ii) Is $6 - 3x + 5x^2 \in F$? EXPLAIN.

QUESTION 2. Given $v_1 = (1, 1, 1, 1), v_2 = (-1, 0, 1, 1), v_3 = (-1, -1, -1, 0)$ are independent in R^4. Find $v_4 \in R^4$ such that $B = \{v_1, v_2, v_3, v_4\}$ is a basis for R^4. Show the work.

QUESTION 3. Are $2 + x, -2, 30 + 7x$ independent in P_2?explain

QUESTION 4. Given $T : R^3 \longrightarrow R_{2\times2}$ such that $T((a_1, a_2, a_3)) = \begin{bmatrix} a_1 & a_1) \\ a_3 & a_2 \end{bmatrix}$ is a linear transformation.

(i) Find the standard matrix representation of T.

(ii) Find a basis for Ker(T) and write Ker(T) as a span.

(iii) Find a basis for Range(T) and write Range(T) as a span.

QUESTION 5. Given $T : P_3 \longrightarrow R$ is a linear transformation such that $T(1) = 2$, $T(x^2 + x) = -5$, and $T(x^2 + 2x + 2) = -8$.

(i) Find $T(x)$ and $T(x^2)$ and $T(5x^2 + 3x + 8)$

(ii) Find the standard matrix representation for T.

(iii) Find a basis for Ker(T) and write Ker(T) as a span.

(iv) Is T ONTO? explain.

QUESTION 6. Let $F = \{a + bx + bx^2 + cx^3 \in P_4 \mid a, b, c, d \in R, a - 2b - d = 0, \text{ and } c - 3b - d = 0\}$.

(i) Show that F is a subspace of P_4.

(ii) Find a basis for F and write F as a span.

QUESTION 7. (**8 points**) Let $D = \{3a + b^2x^2 + 4ax^3 \mid a, b \in R\}$ Is D a subspace of P_4? If NO, explain. If YES, find a basis for D

QUESTION 8. (**15 points**) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 2 & 3 & 4 \\ 2 & 2 & 2 & 2 & 5 & 2 \end{bmatrix}$

(i) Find a basis for $ROW(A)$.

(ii) Find a basis for $Col(A)$

QUESTION 9. (**10 points**) Given $L = \{\begin{bmatrix} 2a & 2a - b \\ b + a & -c \end{bmatrix} \mid a, b, c \in R\}$ is a subspace of $R_{2\times2}$. Find a basis for L.

QUESTION 10. Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$ Given $det(A) = 21.23$ Consider the following system $AX = \begin{bmatrix} 3.2a_2 + a_1 \\ 3.2a_5 + a_4 \\ 3.2a_8 + a_7 \end{bmatrix}$. Solve for x_1, x_2, and x_3.

QUESTION 11. $A = \begin{bmatrix} 1 & 2 & 2 & -4 \\ 0 & 0 & 3 & -2 \\ -1 & 4 & 2 & 2 \\ -1 & -2 & -2 & 8 \end{bmatrix}$ Find the $(2,4)$-entry of $A{-}1$

QUESTION 12. (i) In each question below a vector space V is given, together with a subset $W \subseteq V$. In each case state (with justification) whether or not W is a sub*space* of V. If W is a subspace of V, find a basis for W and the dimension of W.

a. $V = R^2,\ W = \left\{(x,y) \in R^2 |\ x \geq 0 \text{ and } y \geq 0\right\}$.

b. $V = R_{2\times 2},\ W = \{A \in R_{2\times 2} | Rank(A) \leq 1\}$

QUESTION 13. (i) Find a basis for the subspace W where

$$W = \left\{ \begin{bmatrix} a-b+3c & 4a+3b-9c & 2a \\ 8a+2b-6c & 5a & 0 \end{bmatrix} \mid a,b,c \in R \right\}$$

QUESTION 14. (i) Let$A = \begin{bmatrix} 2 & -4 & 0 & -6 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & 2 & 0 & 3 & 6 \end{bmatrix}$. If $T : R^5 \to R^3$ is defined by $T((a_1,a_2,a_3,a_4,a_5)) = A \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$

a. Find a basis for the range of T

b. Find $Ker(T)$ and Write $Ker(T)$ as a span

QUESTION 15. (i) The linear transformation $T : P_3 \to R$ is given by

$$T(p(x)) = \int_{-1}^{1} p(x)dx$$

a. Find a $p(x)$ in P_3 so that $T(p(x)) = 2011$.

b. Find $Ker(T)$ and write Ker(T) as a span.

c. Is T one to one?explain

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